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Investigations of Image Compression using Polynomial Fitting of the Singular Values

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***Abstract*—** An image, though it appears simple, needs numerous pixel values to represent it. If the image may be represented using lesser number of parameters, the image may be easily processed, stored, and transmitted. There are several techniques for compression. For example, singular value decomposition (SVD), Eigen values based analysis, discrete cosine transform, wavelet based transform, etc. Although each of these techniques has been used extensively in literature, SVD has been shown more advantageous. In the present research, the variation of singular values is approximated by a polynomial and at the time of synthesis, singular values are computed from the polynomial whose coefficients are estimated at the time of analysis. The objective is to explore the use of polynomial fitting for representing the whole set of singular values. Because the polynomial equation may be represented using smaller number of coefficients, it may be expected that the proposed technique would reduce the size of the image leading to better compression ratio.

***Keywords*—** Digital images; eigen values; enhancement; principle component analysis; singular value decomposition.

I. Introduction

Images convey more information as compared to phrases used in textual or verbal communication[1]. An image is a function of two real variables, represented as $f(x,y)$. Here, f is the intensity at (x,y) coordinate position, called the pixel [2]. When x, y and intensity values of f are finite discrete values, then the image is called as digital image [3]. The increasing use of images in medicine, education, remote sensing, and entertainment has led to vast image archives that requires effective management and retrieval strategies [4].

Image processing is carried out to enhance an image by applying some mathematical transformation for deriving desired parameters. Input image may be obtained from a still camera, frame of a video clip, or scanned photograph. Usually, image processing systems acquire two dimensional images and apply a specific processing method to enhance it [5]. The use of image processing techniques may improve or distort an image [6]. Image processing may be carried out on geometric figures and surfaces textures using the same technique [7]. Image processing involves several overlapping fields in computer engineering, image denoising, enhancement, and image compression being the mostly employed.

Generally, the images contain noise because of defective devices, inefficient data gathering process, or natural disturbances. All these factors lower the quality of acquired image. Thus, the first step is to denoise the image. An improperly selected method for denoising may further degrade the image by introducing blurring of the image. In the recent years, a good amount of research has been carried out on image denoising using wavelet as it provides an efficient mechanism of denoising [8].

Image enhancement is the process of improving the image parameters so that better input can be provided to the image processing systems. It is used in a wide variety of fields including medical imaging, art studies, forensics, and atmospheric sciences. There are several methods to enhance a digital image without degrading its image quality. These techniques may generally be grouped into two classes: spatial domain methods and frequency domain methods. In spatial domain methods, the pixel values are manipulated to get desired processed image. In frequency domain, all the operations of image enhancement are executed on the Fourier transform of the image. After that, the inverse Fourier transform is performed to obtain the processed image. Wavelet based methods have also been investigated and these methods provide better results as compared to Fourier transform [9]. The enhancement operations may modify the image in terms of brightness, contrast, or the distribution of the grey levels [10]. The enhancement procedure makes pixel values more noticeable. As a result different objects can be easily identified in an image [11].

Image compression is a method to transform and organize the information stored in an image in such a way that it can be easily transmitted with lesser memory and bandwidth [12]. It is performed to represent an image using lesser number of pixels while retaining the picture quality of the image. Image compression process requires two algorithms – the image compression algorithm and the image reconstruction algorithm [13]. There are several techniques for compression. For example, principal component analysis (PCA), singular value decomposition (SVD), Eigen values based analysis, discrete cosine transform, wavelet based transform, etc. Although each of these techniques has been used extensively in literature, PCA and SVD have been more advantageous. Image compression techniques view image as a matrix and then operations are performed on the image matrix. Image compression reduces the redundant pixel values, storage space requirement, while maintaining the quality of image [14]. Most image compression algorithms typically divide an image

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into small blocks and then separately treat each block but SVD compresses an image on the whole [15].

The objective of this research is to investigate the effect of polynomial fitting of singular values on the quality and texture of natural images, particularly trees and their leaves. Section II presents the details of PCA and SVD. The methodology of the experimentation has been explained in Section III. The results obtained using the proposed technique is discussed in Section IV.

II. Principal Component Analysis and Singular Value Decomposition

In the analysis and design of image processing systems, it is necessary to identify the mathematical properties of an image. It is easy to process image using some mathematical operation [16]. A number of techniques are available to process an image mathematically. PCA mainly reduces the number of interrelated variables and retains maximum variations present in the variable set. The first few values hold most of the variations present in the original data [17]. PCA provides a standard for reducing a complex set of variables to a structure of simple variables that can also represent the image. PCA is used in all forms of analysis from neuroscience to computer graphics because it is a simple method of extracting important information from large set of variables [18]. The Eigen value and Eigen vector are determined from a symmetric matrix using PCA [19]. PCA is a successful technique due to the following two important properties:

1. Principal components sequentially contain the maximum variations, thus guaranteeing minimal information loss.
2. Principal components are independent. One principal component can be considered without referring to others [20].

PCA is applied on a matrix for simplification, data reduction, representation, variable selection, organization and prediction of an image. PCA extracts the leading patterns in the matrix [21].

Mathematically, PCA is defined as an orthogonal linear transformation and assumes all its basis vectors as an orthonormal matrix. It is the simplest method of analysing the multivariate data based on the Eigen vector. Its operation can be thought of as revealing the internal structure of the data. For implementing Principal Component Analysis, first calculate the mean of the original data. Then, Subtract off the mean for each dimension. Calculate the covariance matrix. Afterwards, find the Eigen value and Eigen vector of the covariance matrix. Extract diagonal of matrix as vector. Sort the column of the Eigen vector matrix and Eigenvalue matrix in order of decreasing Eigen values. Choose the components and form a feature vector.

$$\text{Feature Vector} = [eig_1\ eig_2\ eig_3 \ldots\ldots\ldots eig_n]$$

Once we have chosen the components (Eigenvectors) that we wish to keep in our data and formed a feature vector, we simply take the transpose of the vector and multiply it on the left of the original data [22].

Similarly, SVD has interesting and attractive algebraic properties, and provides important geometrical and theoretical understandings about linear transformations [23]. SVD has attractive properties and using these properties in various image applications is currently at its infancy [24]. Generally, SVD finds applications in problems involving large matrices, having dimensions up to thousands. SVD provides a numerically reliable estimate of the effective rank of a matrix. The way to find these dependencies is to focus on the singular values that are of a larger magnitude than the measurement error. If there are *r* such singular values, the effective rank of the matrix is found to be *r*. The value of *k* i.e. number of Eigen values for the image varies with application. It is observed that if the value of k chosen is equal to the rank of the image, the processed image is closer to the original image. As the value of *k* decreases, the image quality degrades [25].

Another application of the SVD is to computing the generalised inverse of a matrix. This is very closely related to the linear least squares problem. SVD decomposition is being applied to extract the relevant information about aircrafts and to model each aircraft as a subspace [26]. SVD pseudo conversion method is also being applied to reconstruct an image from projections [27].

Mathematically, Given $A \in R_{m*n}$, a singular value decomposition of A is:

$A = U\Sigma V$, where U_{m*m} and V_{n*n} are orthogonal matrices and Σ is a diagonal matrix having same dimensions as A. The diagonal entries, $\Sigma_{ii} = \sigma_i$, are non-negative and can be arranged in decreasing order; the positive ones are called the singular values of A. The columns of U and V are called left and right singular vectors of A [28].

In matrix form, it is represented as

$$A = [u_1, u_2, \ldots, u_m] \begin{pmatrix} \sigma_1 & 0 & 0 & 0 & \\ 0 & \sigma_2 & 0 & 0 & 0 \\ 0 & 0 & \sigma_3 & 0 & 0 \\ \vdots & & & \ddots & \\ 0 & 0 & 0 & 0 & \sigma_n \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \\ \vdots \\ v_n \end{pmatrix}$$

where $u_1, u_2, \ldots, u_m$ are m*1 column vectors, $v_1, v_2, \ldots, v_n$ are 1*n row vectors and σ_i for i=1,2,……,n are singular vectors of A. The singular values are arranged along the diagonal of Σ in such a way that $\sigma_1 \geq \sigma_2 \geq \ldots\ldots \geq \sigma_n \geq 0$. Also, $\sigma_i = \sqrt{\lambda_i}$ where λ_i for i=1, 2,……, n are Eigen values of A. By applying SVD on an image, the image matrix A is decomposed into 3 different matrices U, V and Σ. After applying SVD, only a few singular values are retained while other singular values are discarded. This follows from the fact that singular values are arranged in descending order on the diagonal of Σ and that first singular value contains the greatest amount of information and subsequent singular values contain decreasing amounts of image information. Thus, the lower singular values containing less important information can be discarded without much image distortion [29].

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III. METHODOLOGY

Investigations were carried out using images of natural objects such as trees and their leaves. Several images involving different types of trees, their collection, and leaves were taken using a high resolution digital camera. The images were analysed using SVD for obtaining singular values. The reconstruction of the processed images was carried out using two techniques. In the first technique, the images were reconstructed using different number of singular values. In the second technique, the singular values were approximated by fitting polynomials of different degrees and the singular values were got from the polynomial for resynthesis. Analysis of the processed images was carried out with respect to quality and texture of the output images.


Fig. 1. Unprocessed image of the leaf.


Fig. 2. Processed image taking first 5 singular values.


Fig. 3. Processed image taking first 10 singular values.


Fig. 4. Processed image taking first 15 singular values.


Fig. 5. Processed image taking first 20 singular values.


Fig. 6. Processed image taking first 25 singular values.


Fig. 7. Processed image of degree 5.


Fig. 8. Processed image of degree 10.


Fig. 9. Processed image of degree 15.


Fig. 10. Processed image of degree 20.


Fig. 11. Processed image of degree 25.

IV. Results

The unprocessed image is shown in figure 1. The synthesized images for different number of singular values are shown in figure 2 to figure 6. For comparison, the images synthesized by using polynomial equations of different degrees are shown in figure 7 to figure 11. As expected, as the number of singular values is increased, the quality also increases. It is clear from the presented images that better quality may be achieved by representing the singular values as polynomial coefficients. The analysis of the images shows that satisfactory quality of the synthesized images may be obtained by using polynomial equations having small degrees as compared to the number of original singular values required to synthesize the image.

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